CANTOR'S ANTIDIAGONAL TEST RETRIEVAL [ALWAYS] SAYS NO

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ABSTRACT. RCDA4: in my previous articles (RCDA2 and RCDA3), I've already shown that Cantor's Diagonal Argument was not valid using formal acceptance used in engineering without looking to why Cantor's Diagonal Argument was failing. This article shows with the help of Hong-Yi Lee's article [4] that, by construction, Cantor's antidiagonal $\overline{d}$, can never be retrieved in the list of real number in the interval $[0, 1)$. Cantor's Antidiagonal Test Retrieval denoted: $T_{CATR} = \left(\overline{d} \in \{a_1, a_2, a_3, \dots\}\right)$ systematically fails $T_{CATR} = 0$, and more seriously than BBC comedy's Little Britain: "Computer says no"[1].
As a consequence based on T_{CATR}'s Shannon Entropy, denoted $E(T_{CATR}$, Cantor's Diagonal Argument does not bring any information (as $E(T_{CATR}) = 0$ bit) to qualify if the one-to-one mapping proposed is bijective or non surjective (i.e. injective-only).

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Part 1. Cantorian Diagonal Argument always fails to retrieve the antidiagonal

1. STANDARD CANTORIAN DIAGONAL: ON NATURAL NUMBERS

Assumption 1.1. *the interval of real numbers* $[0, 1)$ *is countable*

Assumption 1.1 means there is a bijection between the set $\mathbb{N}$ of natural numbers and the interval $[0, 1)$ of real numbers.

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1.1. 1^{st} step of Cantor's Diagonal Argument Under assumption 1.1 Cantor's Diagonal Argument start with an implicit bijective mapping between $\mathbb{N}$ and the real numbers interval $[0, 1))$ expressed in radix r with unspecific positional numbers $a_1, a_2, a_3, \dots$ without particular constraints as follow (reusing some notations in [3]:

$$\begin{aligned} 1 &\leftrightarrow a_1 = 0.a_{11}a_{12}a_{13}\dots \\ 2 &\leftrightarrow a_2 = 0.a_{21}a_{22}a_{23}\dots \\ 3 &\leftrightarrow a_3 = 0.a_{31}a_{32}a_{33}\dots \\ &\vdots \end{aligned} \tag{1.1}$$

1.2. 2^{nd} Cantor Diagonal Argument step: Let $\overline{d} = f(a_1, a_2, a_3, \dots)$ be the antidiagonal formed by the incremented digits (modulo radix r) of the diagonal as follow (reusing [2]):

$$\overline{d} = 0.\bar{d}_1\bar{d}_2\bar{d}_3\dots \text{ with } \bar{d}_i \equiv a_{ii} + 1 \bmod r \;\; \forall i \in \{1, 2, 3, \dots\} \tag{1.2}$$

1.3. Cantor's Antidiagonal Test retrieval definition:

Definition 1.2. Let $T_{CATR}(\cdot)$ be Cantor's Antidiagonal Test retrieval, a logical test to retrieve the antidiagonal defined in equation 1.2 in the list of numbers $a_1, a_2, a_3, \dots$ denoted as:

$$T_{CATR}(a_1, a_2, a_3, \dots) = \left(\overline{d} \in \{a_1, a_2, a_3, \dots\}\right) = \begin{cases} 0 \iff \overline{d} \notin \{a_1, a_2, a_3, \dots\} \\ 1 \iff \overline{d} \in \{a_1, a_2, a_3, \dots\} \end{cases}$$

1.4. Theorem: Cantor's Antidiagonal Test Retrieval always fails

Theorem 1.3. *Cantor's antidiagonal test retrieval, the logical test to retrieve the antidiagonal $\overline{d}$ defined in equation 1.2 always fails:*

$$T_{CATR}(a_1, a_2, a_3, \dots) = 0 \;\; \forall a_i \in [0, 1) \;\; i = (1, 2, 3 \dots)$$

Proof 1.4. Although in his article[4] Hong-Yi Lee did not formulate a proof of a theorem, the proof given here is 90% copied from Hong-Yi Lee's extremely clear formulation above his equation (4): Since equation 1.2 guarantees that for any real number a_i, $\overline{d}$ has one digit $\bar{d}_i$ distinct from the diagonal digits a_{ii} of a_i $i = (1, 2, 3 \dots)$, which ensures that for any real number :

$$a_i \neq \overline{d} \quad i = (1, 2, 3 \dots) \implies \overline{d} \notin \{a_1, a_2, a_3, \dots\} \iff T_{CATR}(a_1, a_2, a_3, \dots) = 0 \tag{1.3}$$

1.4.1. *Cantor's Antidiagonal Test retrieval $T_{CATR}(\cdot)$ does not bring any information (Shannon Entropy=0 bit)*

Theorem 1.5. *Cantor's antidiagonal retrieve test $T_{CATR}(\cdot)$ does not bring any information (as defined by Shannon Entropy):*

$$E\left(T_{CATR}(a_1, a_2, a_3, \dots)\right) = 0 \; bit$$

Proof 1.6. Let's denote p_0 the probability $T_{CATR}(a_1, a_2, a_3, \dots) = 0$
and p_1 the probability $T_{CATR}(a_1, a_2, a_3, \dots) = 1$.
Equation 1.2 $\implies$ $p_0 = 1$ and $p_1 = 0$ therefore Shannon entropy of $T_{CATR}(a_1, a_2, a_3, \dots)$ is given by the following equation:

$$E\left(T_{CATR}(a_1, a_2, a_3, \dots)\right) = p_0 \log_r(\frac{1}{p_0}) + p_1 \log_r(\frac{1}{p_1}) = 1 \log_r(\frac{1}{1}) - 0 \log_r(0) = 0 \; bit \tag{1.4}$$

1.5. Cantor Diagonal Argument result step:

Result 1.7. *Cantor find that the antidiagonal* $\overline{d}$ *is not accounted for in the sequence* $\{a_1, a_2, a_3, \ldots\}$ *mapped to natural numbers:* $\{1, 2, 3, \ldots\}$. *Which is perfectly aligned with theorem 1.3*

$$T_{CATR}(a_1, a_2, a_3, \ldots) = 0$$

Unfortunately, without realising that the antidiagonal can never be retrieved as shown in Theorem 1.3 and that Cantor's Antidiagonal Test Retrieval does not brings any information, **Cantor wrongly implies a non surjective one-to-one mapping**, *however:*

$$(1.5) \quad T_{CATR}(a_1, a_2, a_3, \ldots) \text{ always } 0 \;\not\Longrightarrow\; \text{the mapping between } [0,1) \text{ and } \mathbb{N} \text{ is non surjective}$$

1.6. Cantor's Diagonal Argument wrong conclusions:

without knowing the result 1.5, Cantor take the wrong conclusions: the assumption 1.1 the interval $[0, 1)$ of real numbers is countable is considered by Cantor as wrong and Cantor believes CDA shows the real numbers in the interval $[0, 1)$ are more numerous than natural numbers in set $\mathbb{N}$:
Cantor final wrong conclusion it therefore the real numbers interval $[0, 1)$ **is uncountable** which can also be expressed as:

$$\big|[0,1)\big| > |\mathbb{N}|$$

Part 2. Conclusion

This transfinite sceptic article shows that the Cantor's Antidiagonal Retrieval Test is guaranteed to fail by construction and therefore it does not bring any information in Shannon Entropy sense. Therefore all conclusions taken by Cantor are meaningless.

References

[1] Little Britain, Lucas & Walliams: *"Computer says no", BBC comedy*.
`https://youtu.be/x0YGZPycMEU?si=tCaQRwNaNKXiWKsP`

[2] Wolfgang Mückenheim: *Transfinity: A Source Book, 2.2.3 Cantor's second uncountability proof p32*.
`https://www.hs-augsburg.de/~mueckenh/Transfinity/Transfinity/pdf`

[3] Hong-Yi Lee: *A Rigorous Examination on Cantor's Diagonal Argument, Now write the above real numbers as (p:1)*.
`https://vixra.org/pdf/2106.0160v1.pdf`

[4] Hong-Yi Lee: *A Rigorous Examination on Cantor's Diagonal Argument (p:2)*,
Since eq.(3) guarantees that for any real number a_i, b has one decimal place, b_i, which is different from the i-th decimal place a_{ii} of the real number a_i, which ensures that for any real number a_i,

$$a_i \neq b \;\; (i = 1, 2, 3, \ldots) \qquad (4)$$

`https://vixra.org/pdf/2106.0160v1.pdf`

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